Error Control Coding - From Theory to Near-Capacity Implementation

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Nera Research

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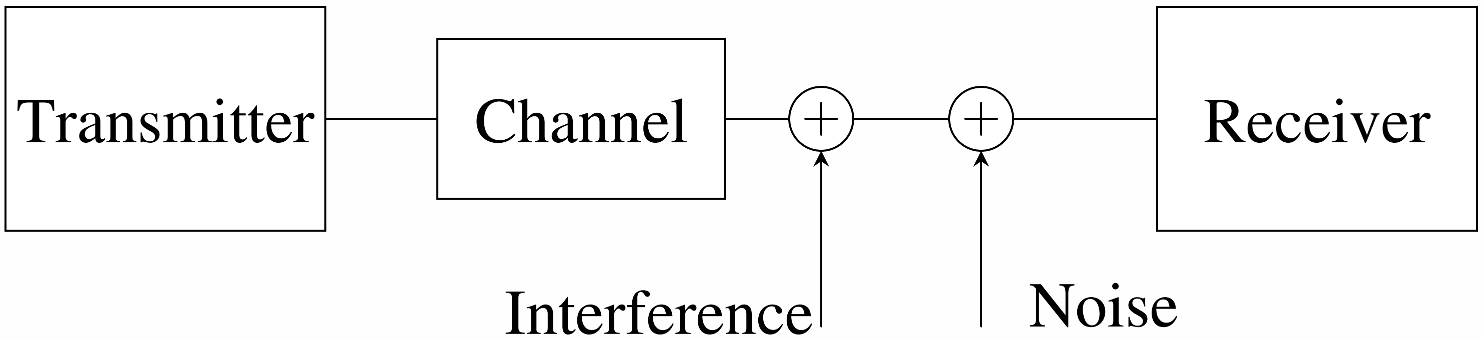
Outline

- Why error control?
- What is possible with error control coding?
- Traditional Coding Schemes
- New Capacity Approaching Schemes
- Coding for High Spectral Efficiency
- Applications - including Nera Products
- Implementation Aspects

Why Error Control Coding

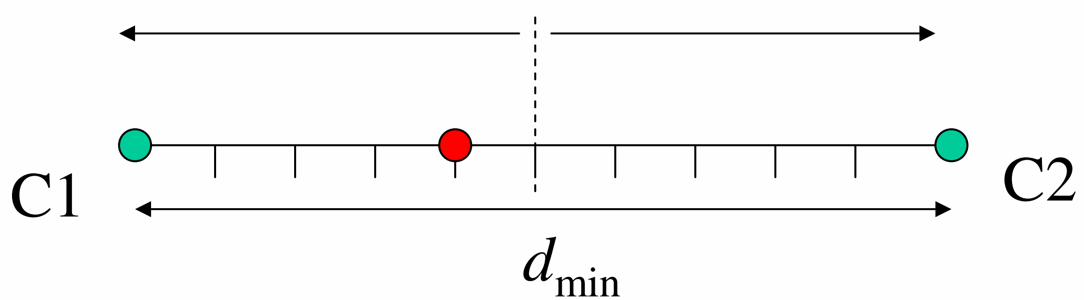


- Thermal noise
- Channel: Fading and inter-symbol interference
- Interference:
  - from system itself and other systems
  - May be combated by channel coding (there are other ways also)

Principle of error control

- Add redundancy to enable
  - Error detection
  - Error correction
  - or both

- d_{min} =10, correct 4 errors, detect d_{min}-1 errors



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Some Theory

Shannon Limit

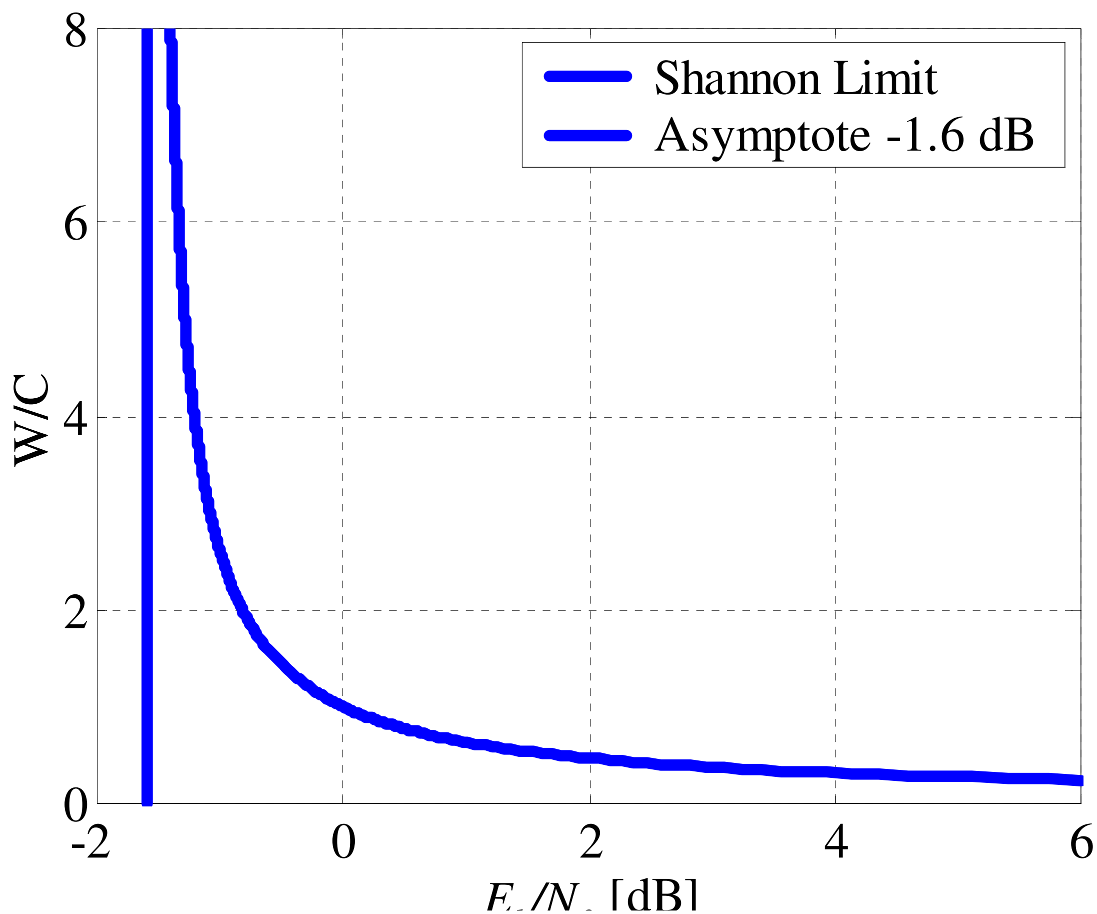
$$C = W \log_2\left(1 + \frac{S}{N}\right)$$

$$\Leftrightarrow \frac{E_b}{N_0} = \frac{W}{C}\left(2^{C/W} - 1\right)$$

$$\boxed{W \to \infty \Rightarrow E_b / N_0 \to -1.6\,\text{dB}}$$

No limit on input, AWGN channel

W/C versus E_b/N_0



- No limit on input
- AWGN channel
- To reach the limit:
  - infinite bandwidth
  - infinite code word length

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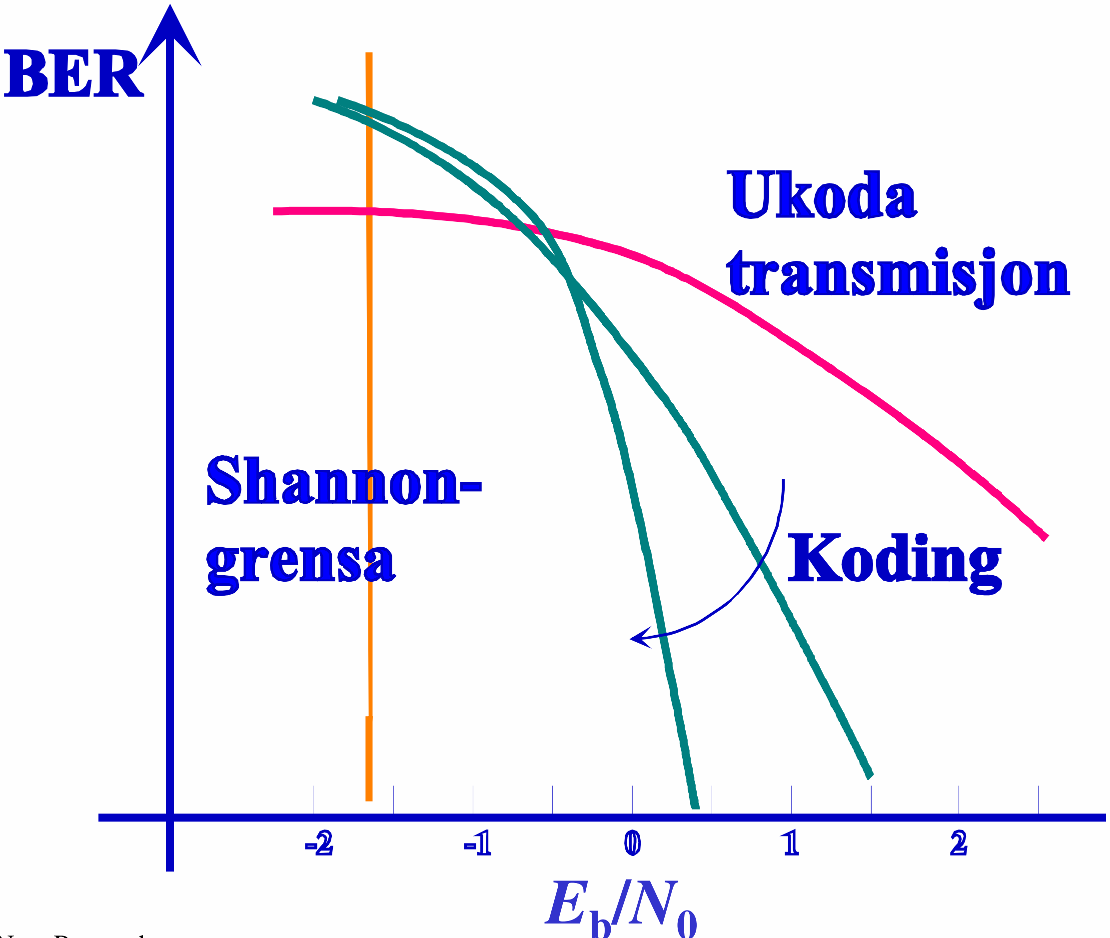
For a Practical Application

- For a practical implementation:
  - code rate $R_c > 0$
  - limits on input (given by modulation)
  - finite code word length

  $\Rightarrow$ -1.6 dB limit - will not be achieved
- Practical Limit - for
  - code rate or bit/s/Hz
  - modulation type
  - code word length

Effect of Coding



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Capacity for given Code Rate

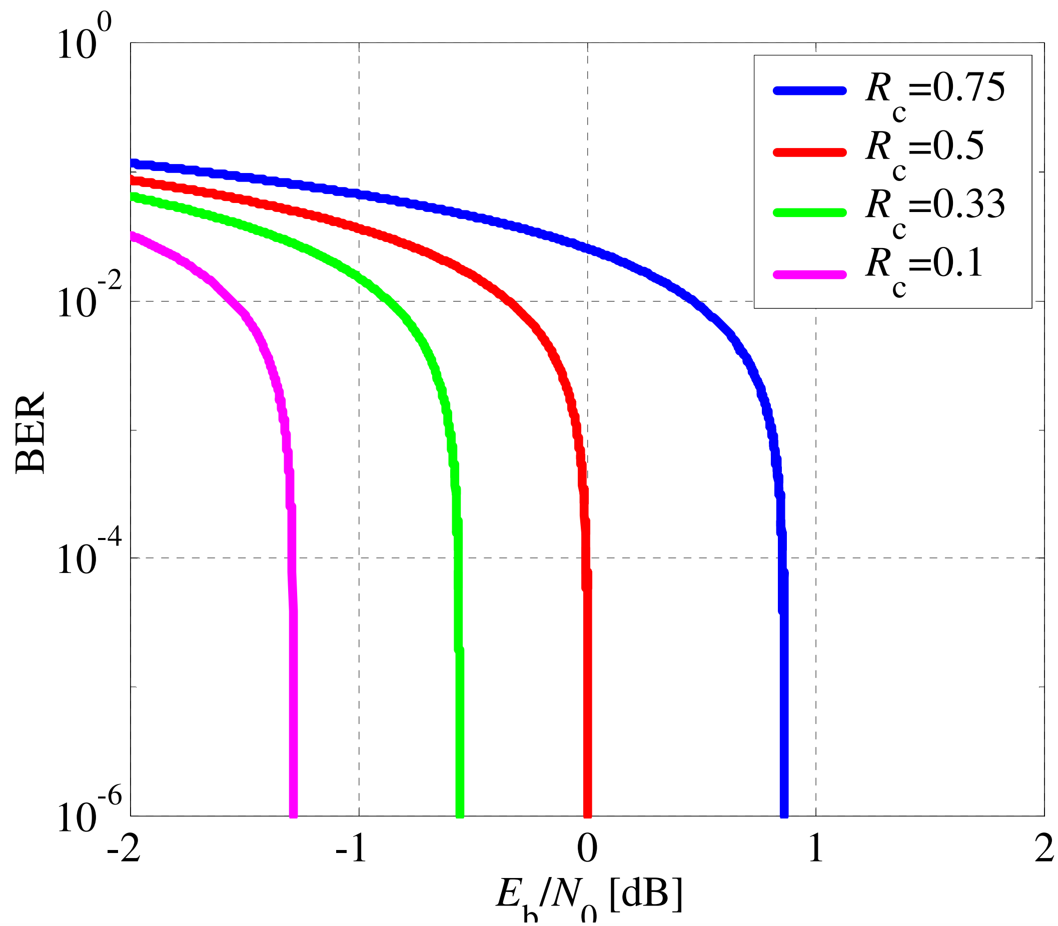
$$C = \frac{1}{2}\log_2(1+\frac{2R_c E_b}{N_0})$$

$$R_c \leq \frac{C}{1-H_b(e)}$$

From the converse
to the coding theorem

Find (P_b, E_b/N_0) pair $\Longrightarrow$ Limit for given code rate

Capacity for different code rates



Unconstrained AWGN Channel

Capacity as function of code rate - binary input (1)

Example: Binary-input (+/- A), AWGN channel

$$C = \frac{1}{2}\int_{-\infty}^{\infty} p(y \mid A)\log_2 \frac{p(y \mid A)}{p(y)}\,dy + \frac{1}{2}\int_{-\infty}^{\infty} p(y \mid -A)\log_2 \frac{p(y \mid -A)}{p(y)}\,dy$$

Constrained AWGN Channel, (From Proakis)

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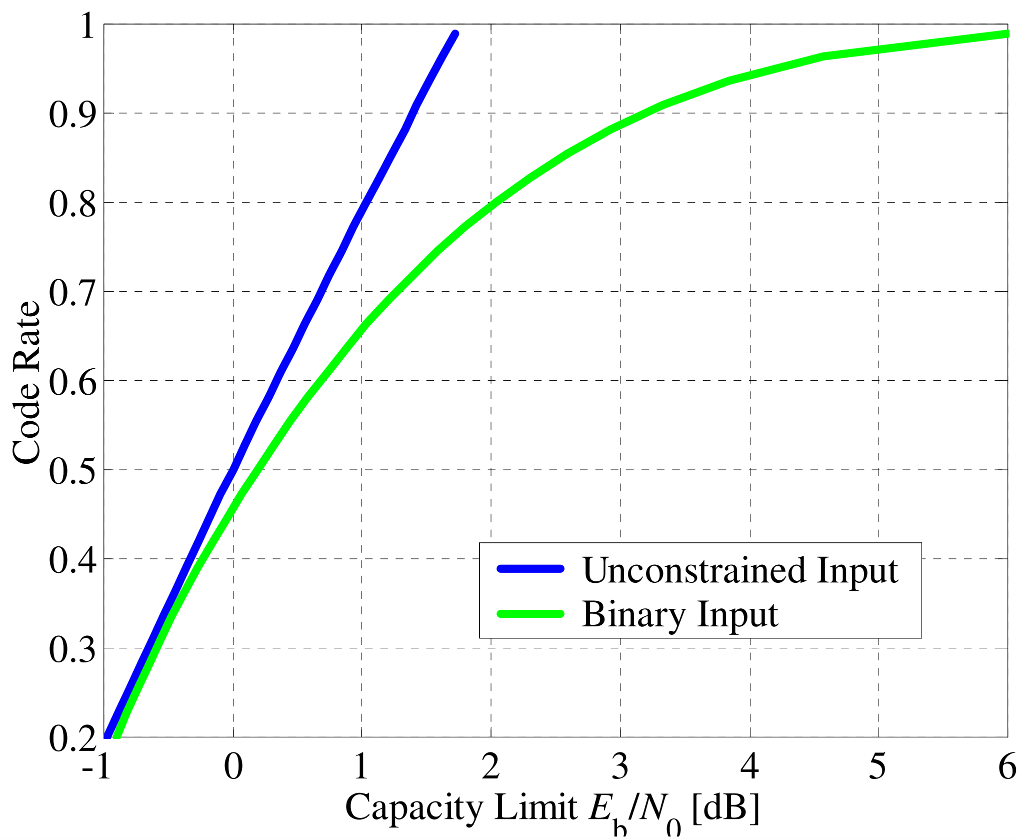
Capacity as function of code rate - binary input (2)



Theoretical limit for given spectral efficiency

- Limited bandwidth, W
- Data rate R (bits/s)
- Shannons capasity can be expressed as:

$$\frac{E_b}{N_0} \geq \frac{2^{\eta} - 1}{\eta}$$

$\eta = \frac{R}{W}$ is spektral-efficiency in bit/s/Hz

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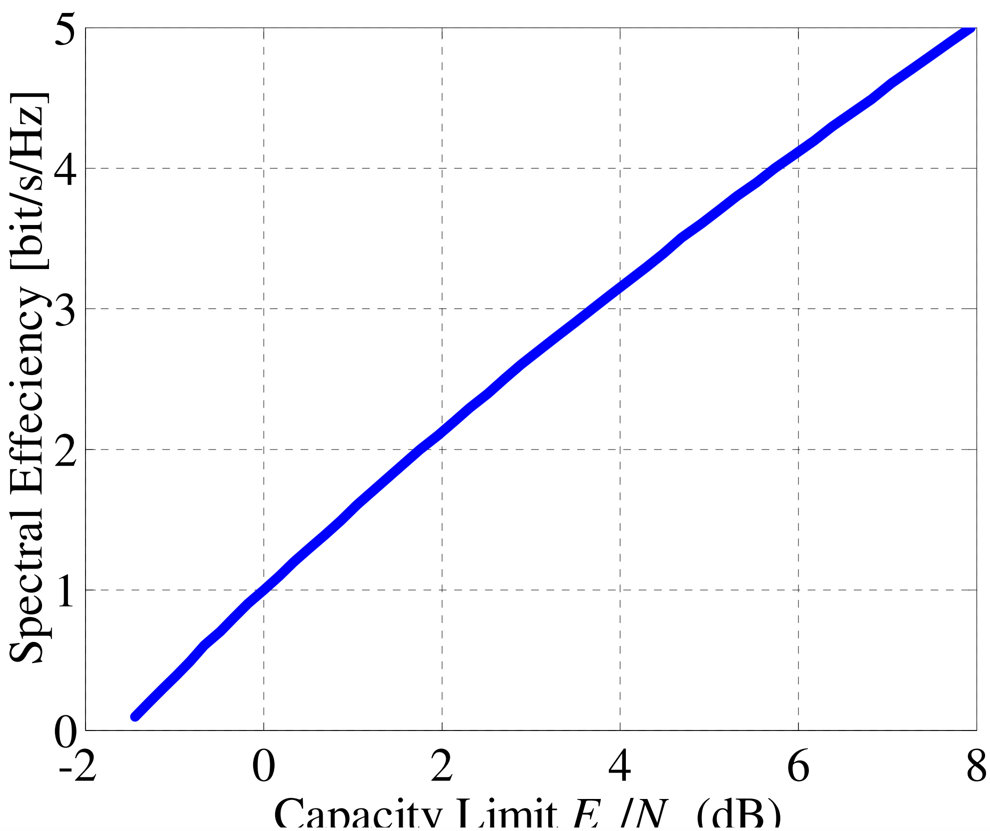
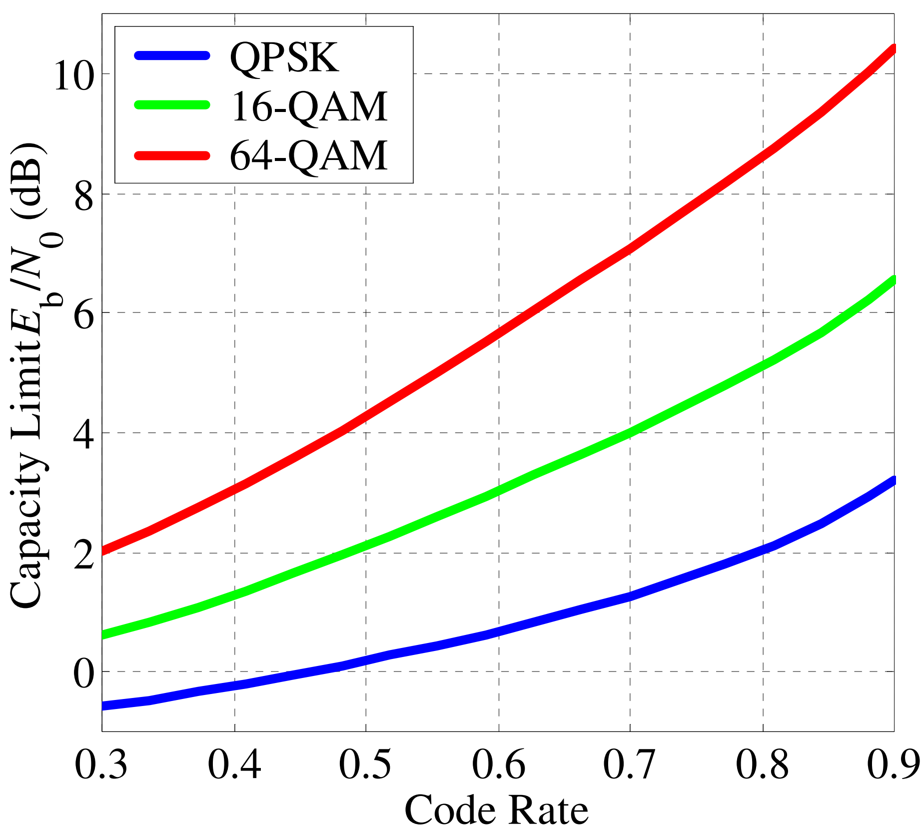
Theoretical limit for given spectral efficiency (plot)



Unconstrained
AWGN
Channel

Larger Modulation Alphabets-Higher Spectral Efficiency



Constrained Additive Gaussian Channel

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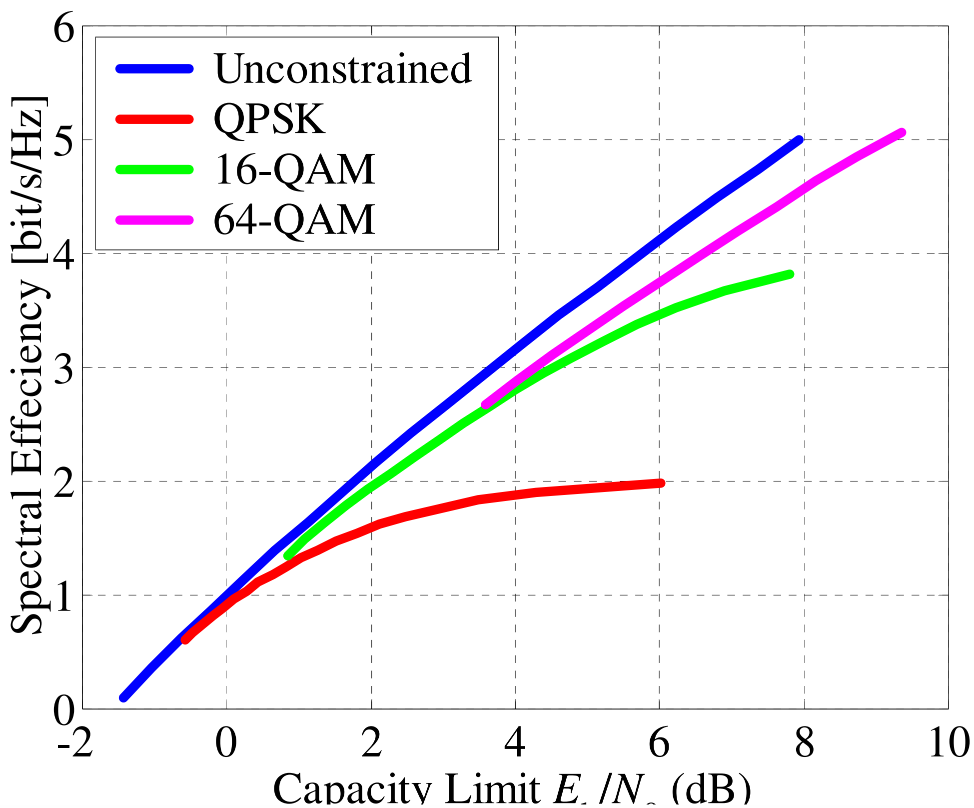
Capacity as function of spectral efficiency

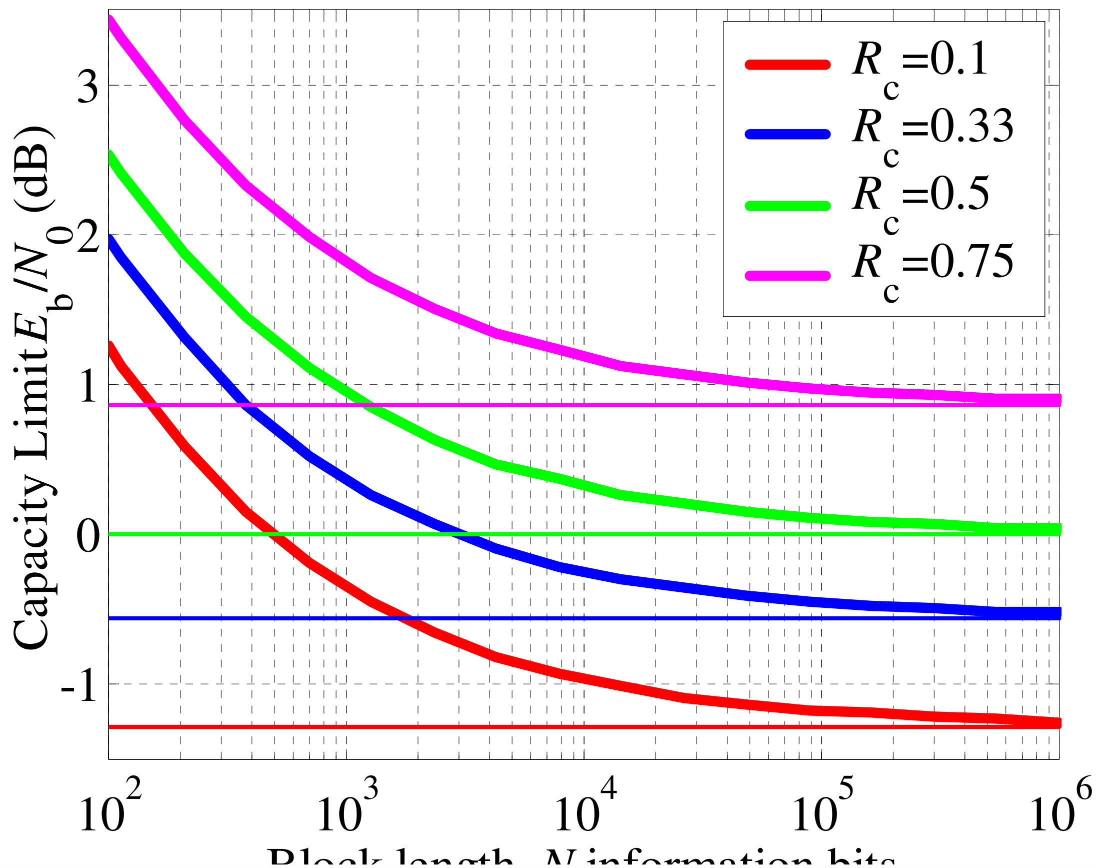


Constrained Additive Gaussian Channel

Calculation:

$\eta = \log_2(M) * R_c$

Capacity as function of Packet Length



Based on paper: Code Performance as Function of Block Size, av Dolinar, Divsalar & Simon

Expression given as function of N, R_c and P_w (Word Error Probability)

Traditional Coding Schemes

Basic Terms

- Code rate

$$R = \frac{k}{n} = \frac{\#Information \quad symbols}{\#Code \quad word \quad symbols}$$

- Soft Decisions
  - Principle idea (more or less correct/wrong)
  - Erasures decoding (3 level soft decision - very simple)
  - 8 level soft decision - often used in many coding schemes
  - Soft decision gain: ca 2 dB for AWGN channels
  - 7-9 dB possible for many fading channels

Traditional Coding Schemes

- Block Codes
  - Long Codes
  - Hard desisjon
  - Good at low bit error rates (10^{-4} and down)
  - Many types
    - Ex. Hamming, Golay, Reed-Muller, Reed Solomon, BCH, etc
  - Used for instance in CD players (RS), Intelsat (RS), Voyager (Golay)
- Convolutional Codes
  - Short Codes
  - Soft desisjon (Viterbi)
  - Good at high bit-error rates (10^{-3} - 10^{-4})
  - Used in Inmarsat, 3G and various wireless systems
  - Long constraint length codes with sequential decoding

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Basic Block Code

- Operates on one block at a time

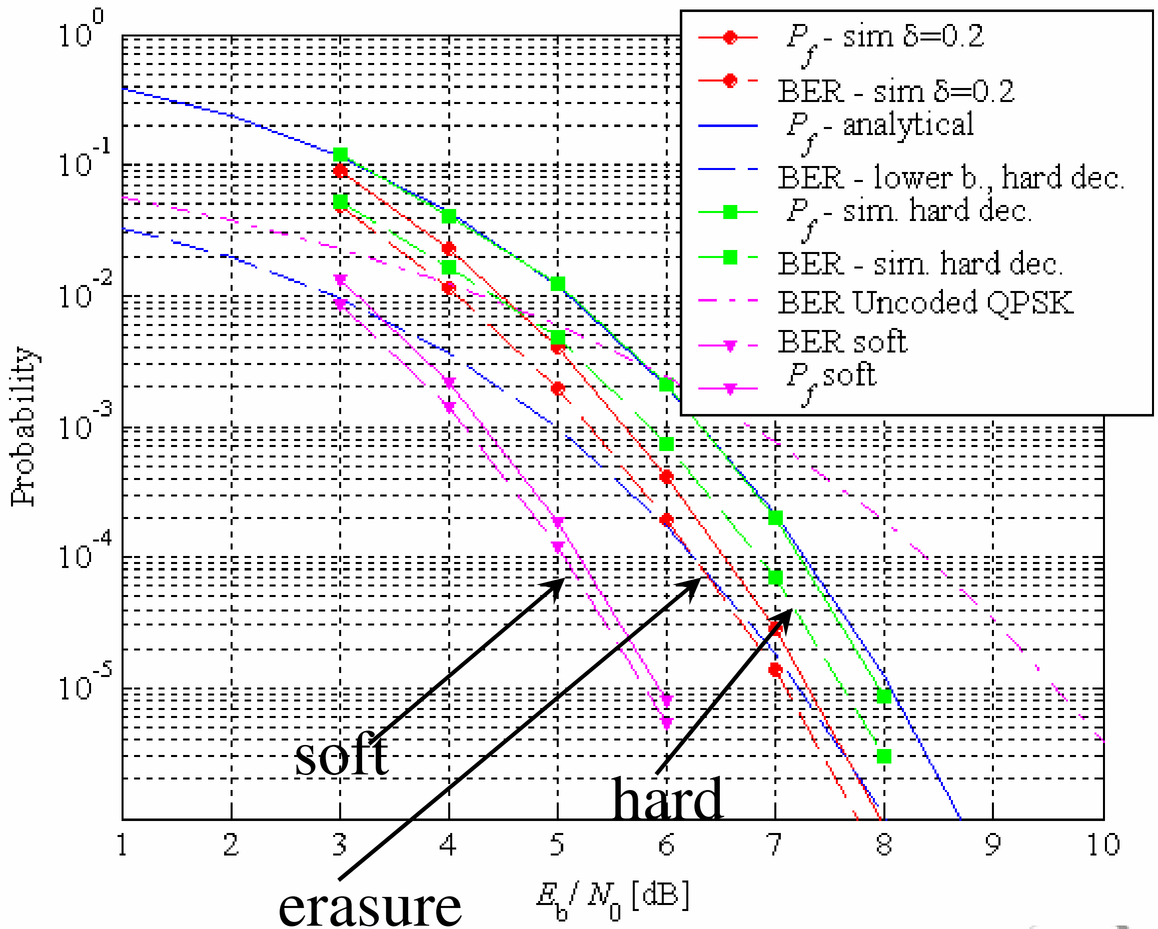
$$c = mG$$

  - Algebraic decoding (often)
  - Soft decision decoding - high complexity
  - Typically good performance at high code rates

Performance Golay Code

- Extended Golay Code (24,12)
- Hard decision
- Erasure decoding
- Soft decoding using Chase algorithm

- Used in WLL product
- Low complexity decoding



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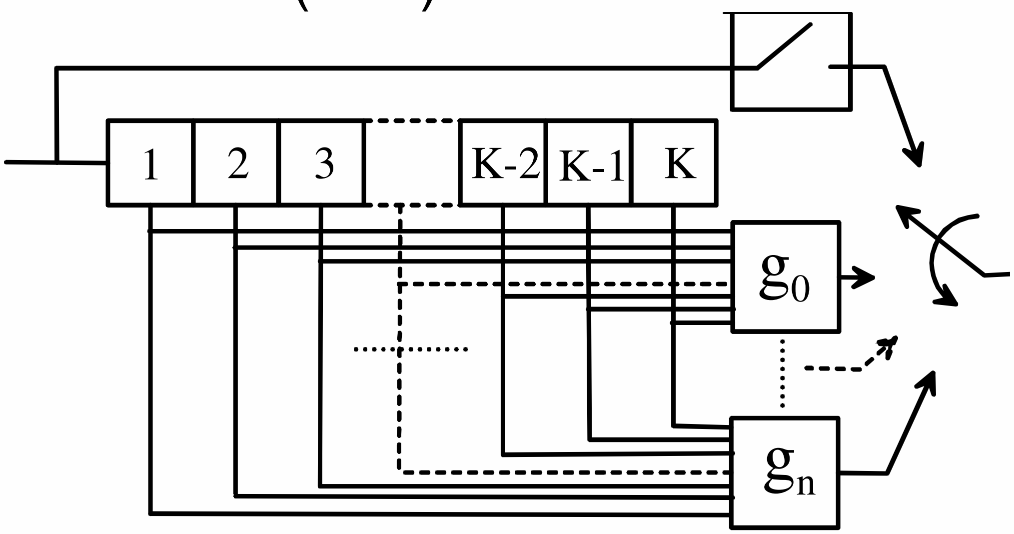
Convolutional Codes

- Described by trellis
- Maximum Likelihood Decoding (ML) by Viterbi algorithm
- Soft Decision decoding easily implemented
- Used in very many applications, Deep Space, SatCom, various wireless systems like 3G
- Rate 1/(n+1) encoder:



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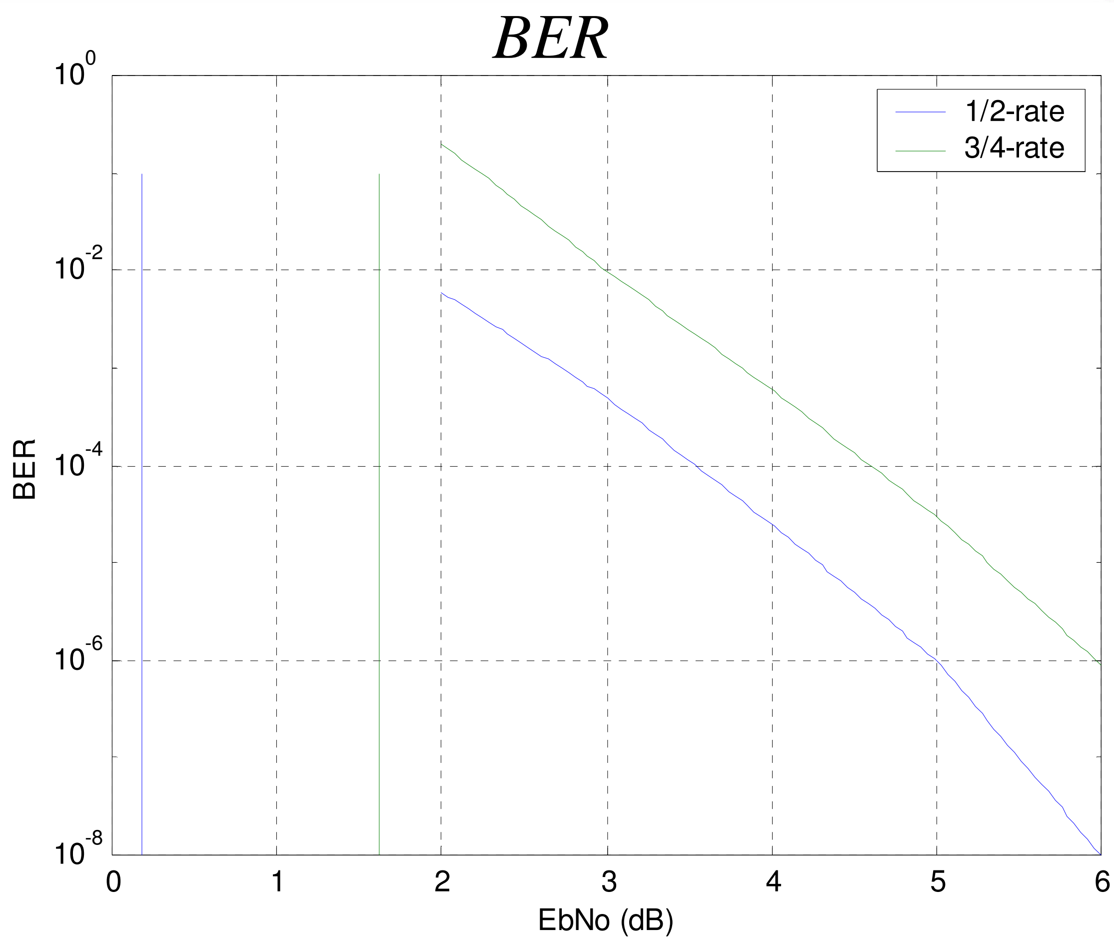
Convolutional Code - BER

- K=7 Feed Forward Code
- MFD (and ODS)
- Viterbi Decoding
- Rate 3/4 obtained by puncturing rate 1/2



E_b/N_0

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Foldningskode - Tap

Koderate	Yting ved 10^{-6}	Kapasitet Constrained Binary Input	Tap pga avgrensa blokklengde	Tap til Constrained input Grense
½	5.0	0.2	??	4.8
¾	6.0	1.6	??	4.4

- Implementert i ASIC
- Relativt stort tap til kapasitetsgrensa
- Blokklengde - vanskelig å definere

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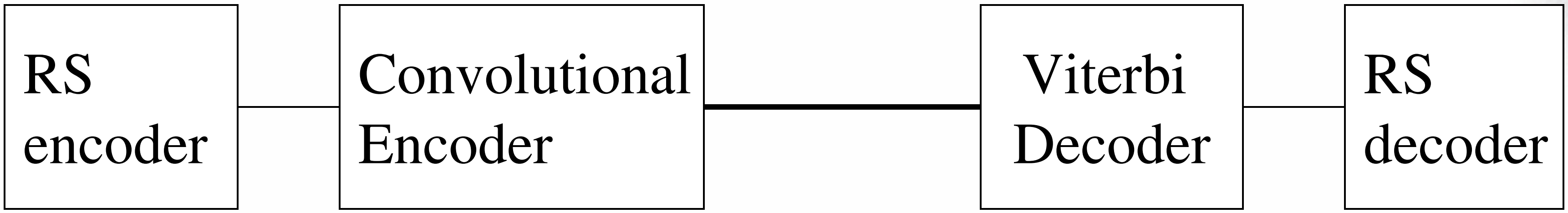
Concatenated Coding



- Traditional scheme for very low error rate
- Typically RS + Convolutional
- Viterbi - soft decisions
- RS burst error correction
- Viterbi gives moderate BER where RS takes over and brings it further down
- Long Code with practical complexity

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Alternative to Concatenation

- Use a very long constraint length (*K*) convolutional code
- Typically - *K*=30-50
- Error rate decreases exponentially with constraint length
- Viterbi decoding:
  - $2^{30} \sim 10^{9}$ states
- NOT IMPLEMENTABLE

- Solution: Sequential Decoding
  - Limited but smart search
  - Search the most likely paths
  - Complexity depends on channel conditions
  - Can calculate conditions for when decoding is possible

Sekvensiell dekoding

Koderate	Pareto eksponent lik 1 ved:	Kapasitet Constrained Binary Input	Tap pga avgrensa blokklengde	Tap til Constrained Grense
½	2.2 dB	0.2 dB	??	2 dB
¾	3.7 dB	1.6 dB	??	2.1 dB

- K=36, R=1/2 Implementert i DSP
- Problem: Overflyt i buffere

Anvendelser i Nera Produkter

- Inmarsat M&B
  - Foldningskode K=7 (1/2 og 3/4-rate)
  - Sekvensiell dekoding K=36 (1/2 rate)
- Nera World Communicator
  - Turbo 1/2-rate med 16-QAM
- DVB-RCS
  - Foldningskode og Reed-Solomon (255,239,8)
  - Turbo
- BGAN (next generation mobile satcom terminals)
  - Turbo 16 tilstander

New iterative capacity approaching schemes

Turbo Codes

ICC 1993 - Presentation of Turbo Codes
- a huge step towards the Shannon limit

- Convolutional Turbo:
  - Parallel Concatenated Convolutional (PCC) codes
  - Serially Concatenated Convolutional (SCC) Codes
  - Separated by interleaver
  - Soft in and soft out, iterative decoding
- Block based Turbo:
  - Turbo Product Codes (TPC)
  - PA-Codes
  - LDPC Codes
  - Soft decision, iterative decoding

Long codes, soft decision, iterative decoding

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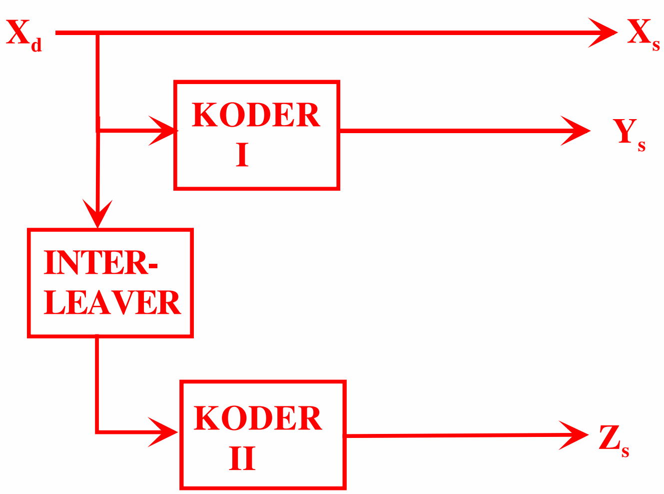
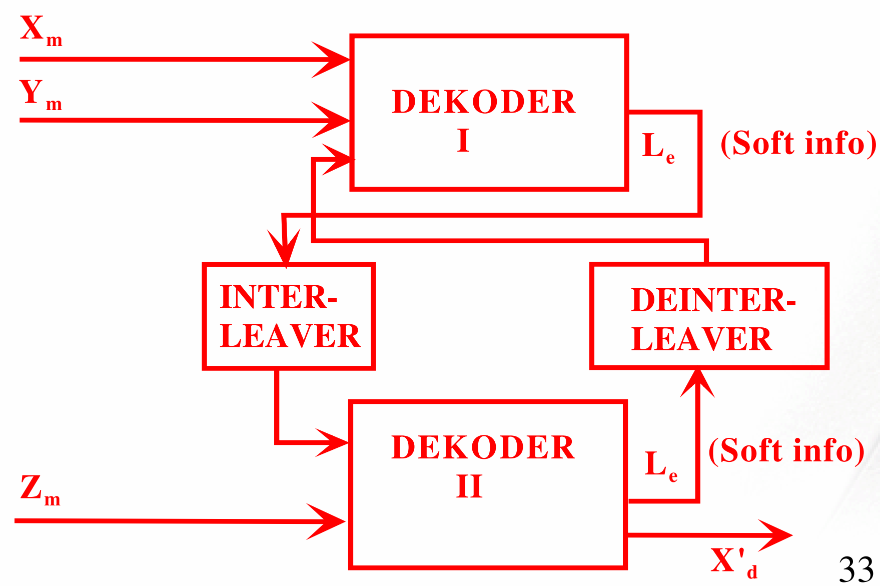
PCC Turbo Principle

- Encoder
  - Recursive convolutional encoders
  - Interleaver essential
- Decoder
  - Soft in soft out
  - Many iterations

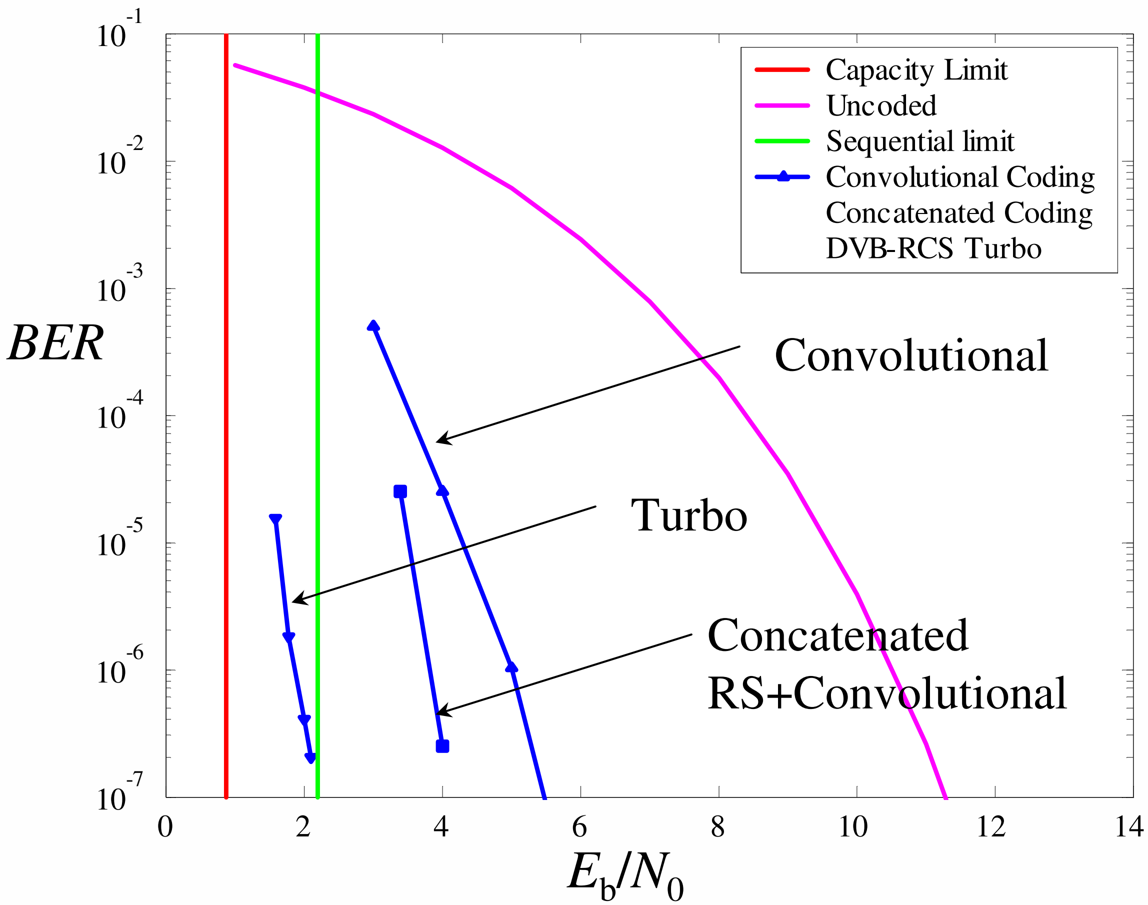




PCC codes - properties

- The ”original” Turbo code as presented by Berrou in 1993
- Good overall performance, also at 10^{-3}
- Good convergence of iterative decoder (few iterations necessary)
- Used in:
  - BGAN (16-states)
  - UMTS (8-states)
  - DVB-RCS (8-states, duo-binary)
- The above codes must be *punctured* to get high rate => affects BER performance

Performance Example



- MPEG block lengths (188 bytes)
- R=1/2

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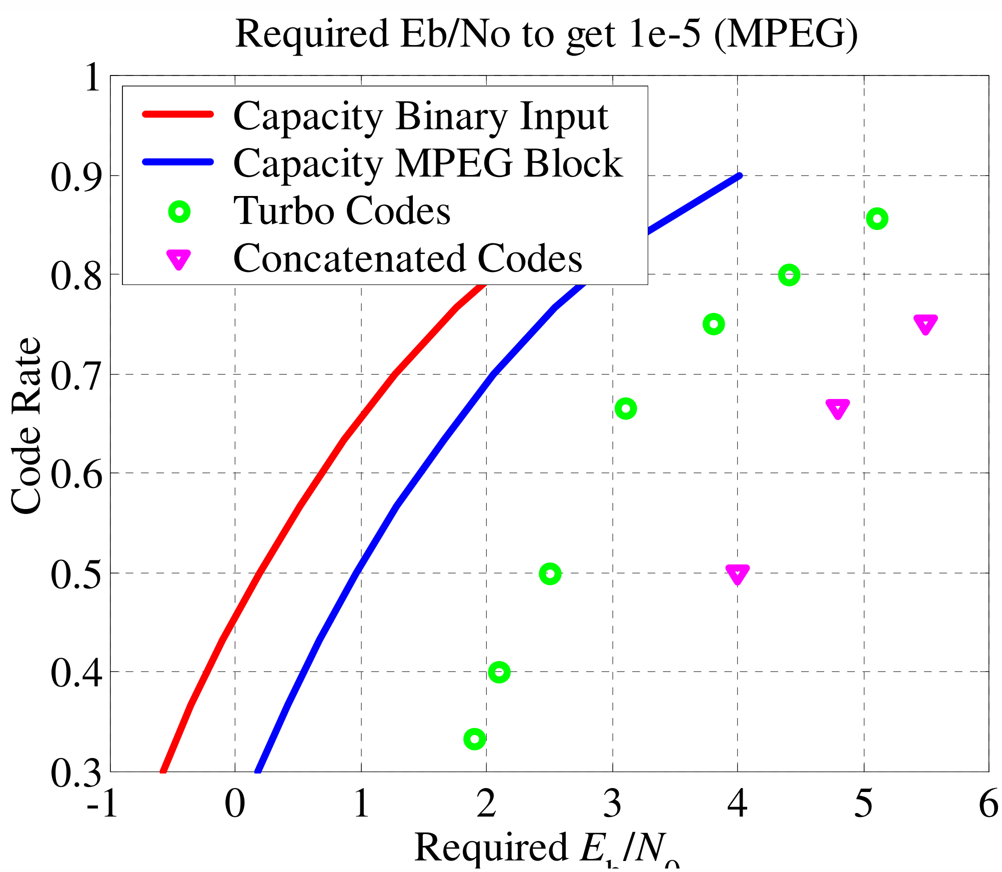
DVB-RCS



Code Performance Comparison for MPEG (1 packet) and PER=10^{-5}

Iterative decoding - Block Codes

- Turbo Product Codes (TPC)
- Product Accumulate Codes
- Low Density Parity Check Codes (LDPC)

- Nera: Theoretical Studies and Simulations
- Interesting because:
  - Typically better than convolutional based Turbo codes at high code rates
  - Fewer patents
  - TPC included in *IEEE*802.16 standard
  - PA codes - low complexity
  - LDPC has “World Record” 0.005 dB from Shannon
  - Complexity -> high-capacity systems

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Turbo Product Codes (TPC)

- Two (or more) block codes concatenated
- Soft in soft out, iterative decoding
- Component codes - e.g. BCH

d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	p_{16}	p_{17}
d_{21}	d_{22}	d_{23}	d_{24}	d_{25}	p_{26}	p_{27}
d_{31}	d_{32}	d_{33}	d_{34}	d_{35}	p_{36}	p_{37}
p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	p_{46}	p_{47}

Ytelse TPC

- Simulert med AWGN, QPSK and 16QAM
- Ytelse ved 10^{-5} BER

(TPC) Koderate	Ytelse ved 10^{-5}	Kapasitet Constrained Binary Input	Blokklengde (kodet)	Ekstra tap Blokklengde	Tap til Praktisk Grense
0.66	3.0 dB	1.2 dB	1024 (32x32)	1.0 dB	0.8 dB
0.79	3.3 dB	2.0 dB	4096 (64x64)	0.5 dB	0.8 dB
		16-QAM			
0.79	6.8 dB	5.1 dB	4096 (64x64)	0.5 dB	1.3 dB

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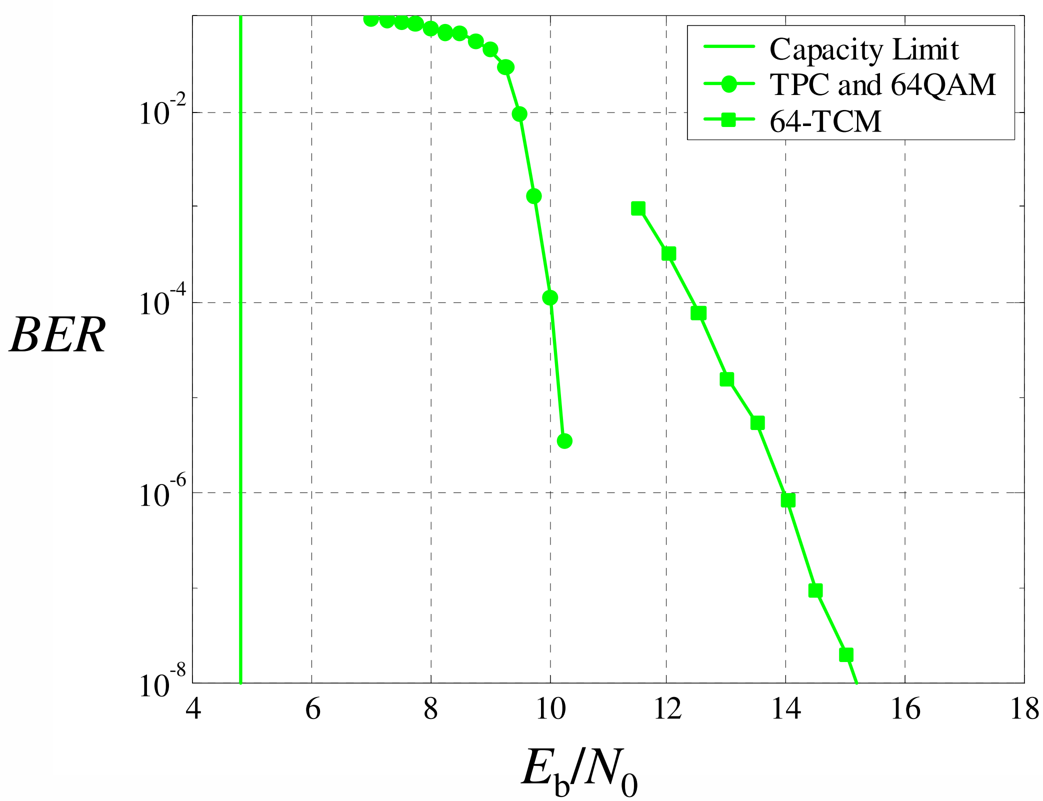
TPC Performance vs TCM

TCM and TPC with 64 QAM, comparable spectral efficiency ((64,57) TPC code)



- Good at high code rates
- Interesting for radio link systems
- In *IEEE*802.16 standard

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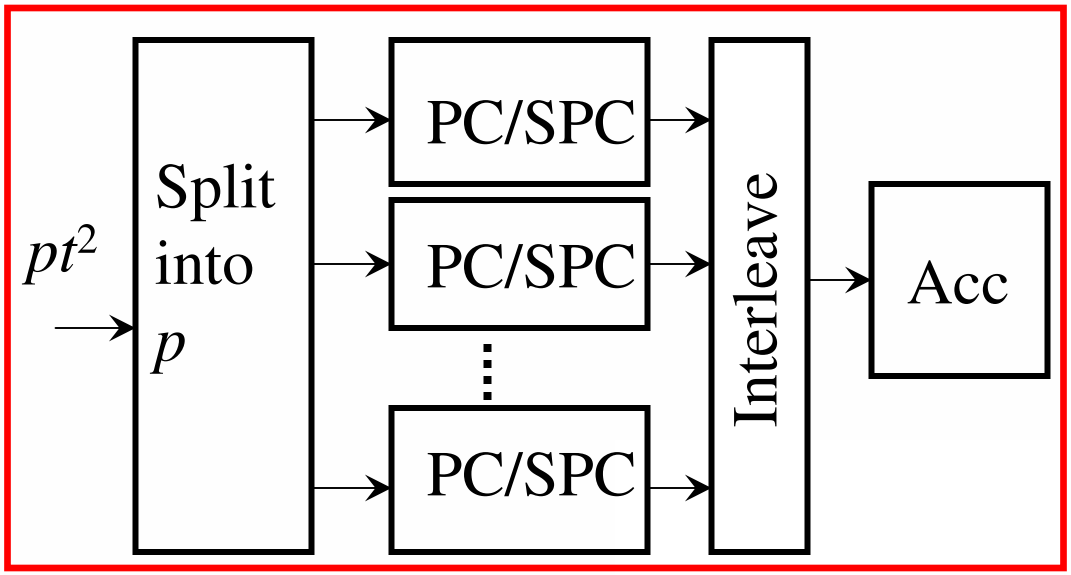
Product Accumulate Codes



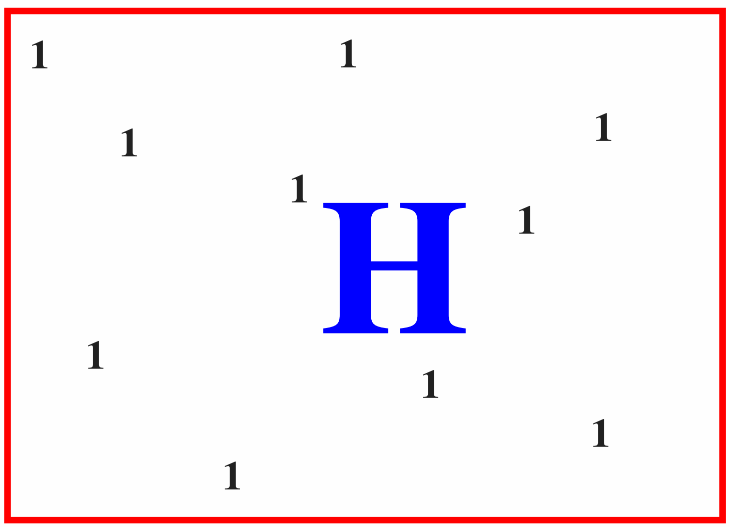
PA code - properties

- A low complexity scheme, presented in 2001
- PA-codes have good overall performance
- No puncturing, assumed to have better performance at low BER
- Few/no patents
- Slower convergence than PCC
  - PA-I
  - PA-II
  - Generalised PA- codes (for lower rates)

Low Density Parity Check (LDPC) Codes

- Block Code
- Iterative decoding

$$c = mG \qquad GH^T = 0$$



LDPC codes -properties

- Proposed by Gallager i 1963
- Good performance, especially for high code rates and long block lengths
- 0.0045 dB from Shannon limit!!
- Low decoder complexity
- Few (no?) patents
- Disadvantages:
  - Some error flattening for some codes
  - Much ongoing research
  - Implementation attempts started

Ytelse - LDPC koder

- Simulert ytelse, BPSK/QPSK, AWGN
- ATM og MPEG blokker
- Regulære LDPC koder, BER=10^{-5}

(LDPC) Koderate	Ytelse ved 10^{-5}	Kapasitet Constrained Binary Input	Blokklengde (kodet)	Ekstra tap Blokklengde	Tap til Praktisk Grense
1/2	1.8 dB	0.2 dB	1504 MPEG	0.7 dB	0.9 dB
4/5	3.7 dB	1.6 dB	1504 MPEG	0.7 dB	1.4 dB
1/2	2.8 dB	0.2 dB	424 ATM	1.2 dB	1.4 dB
4/5	4.5 dB	1.6 dB	424 ATM	1.2 dB	1.7 dB

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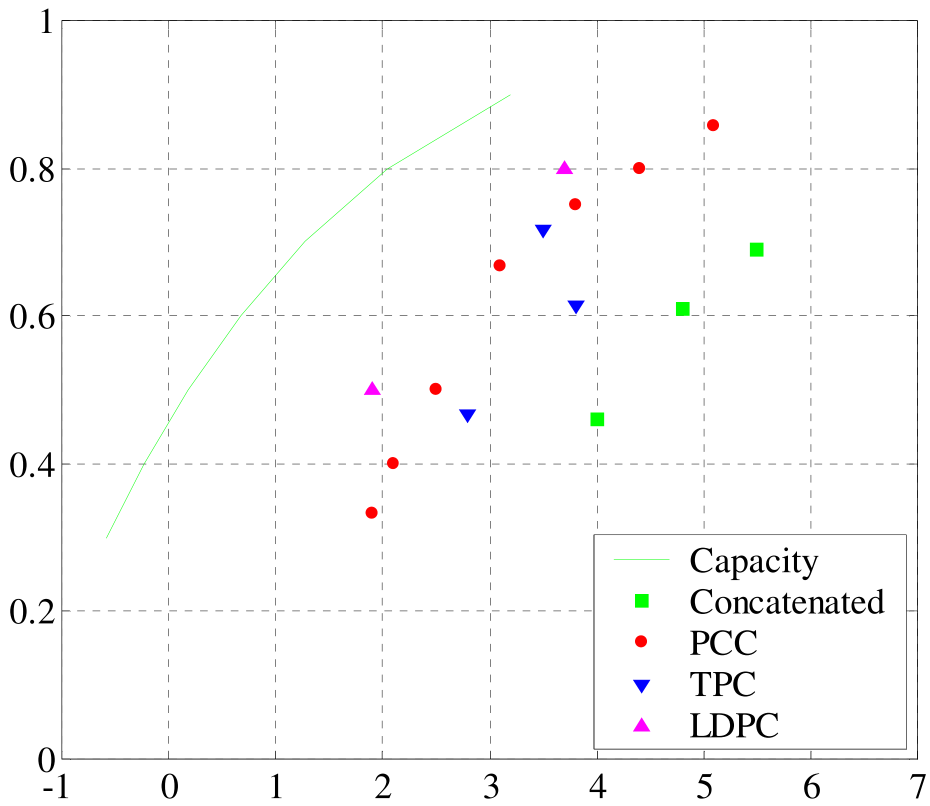
Performance Example LDPC Codes

Spectral Efficiency



Required E_b/N_0

- PER=10^{-5}
- BER=10^{-5} for LDPC
- MPEG packets

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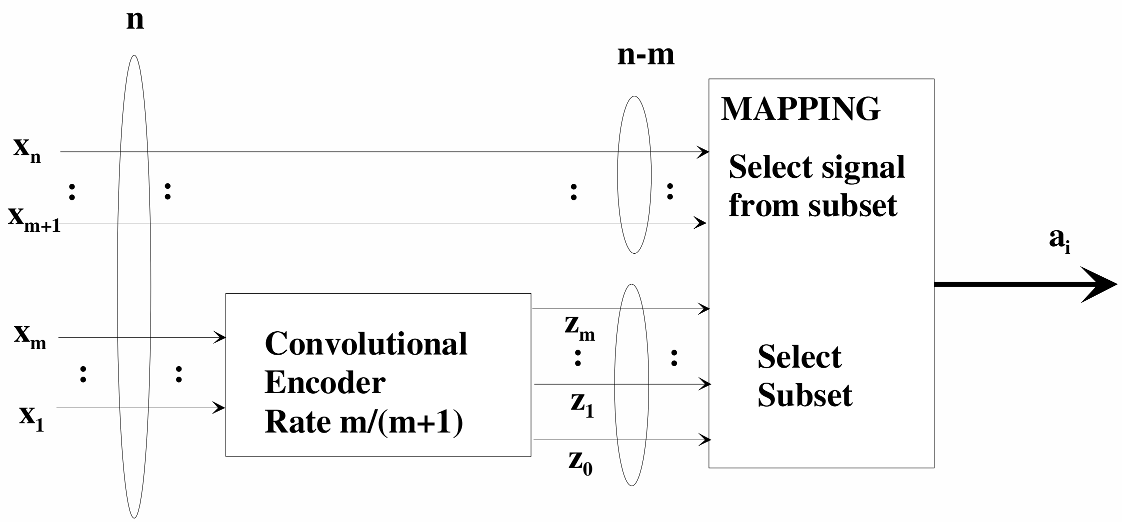
Mapping to higher order constellations

TCM



- Optimised mapping necessary in order to achieve coding gain
- Effective "block length" of code is short

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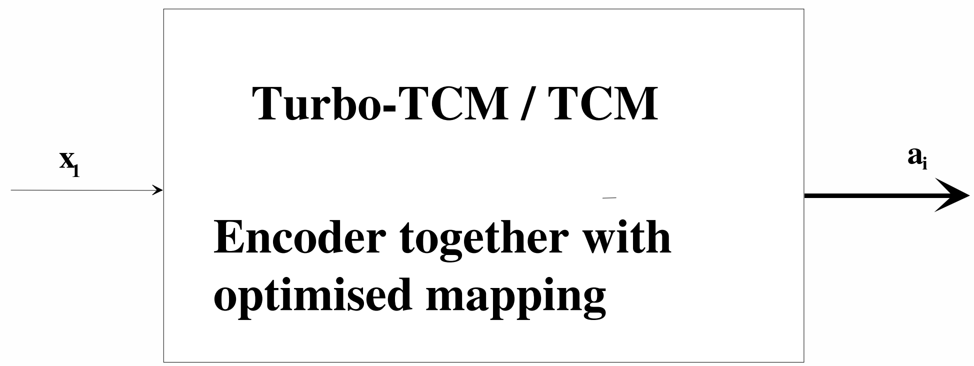
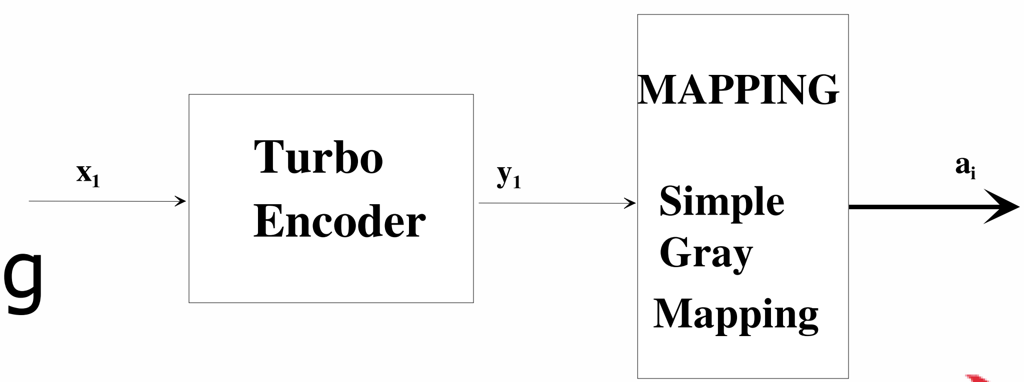
Mapping to higher order constellations

Turbo-TCM / TCM



Encoder + Gray mapping



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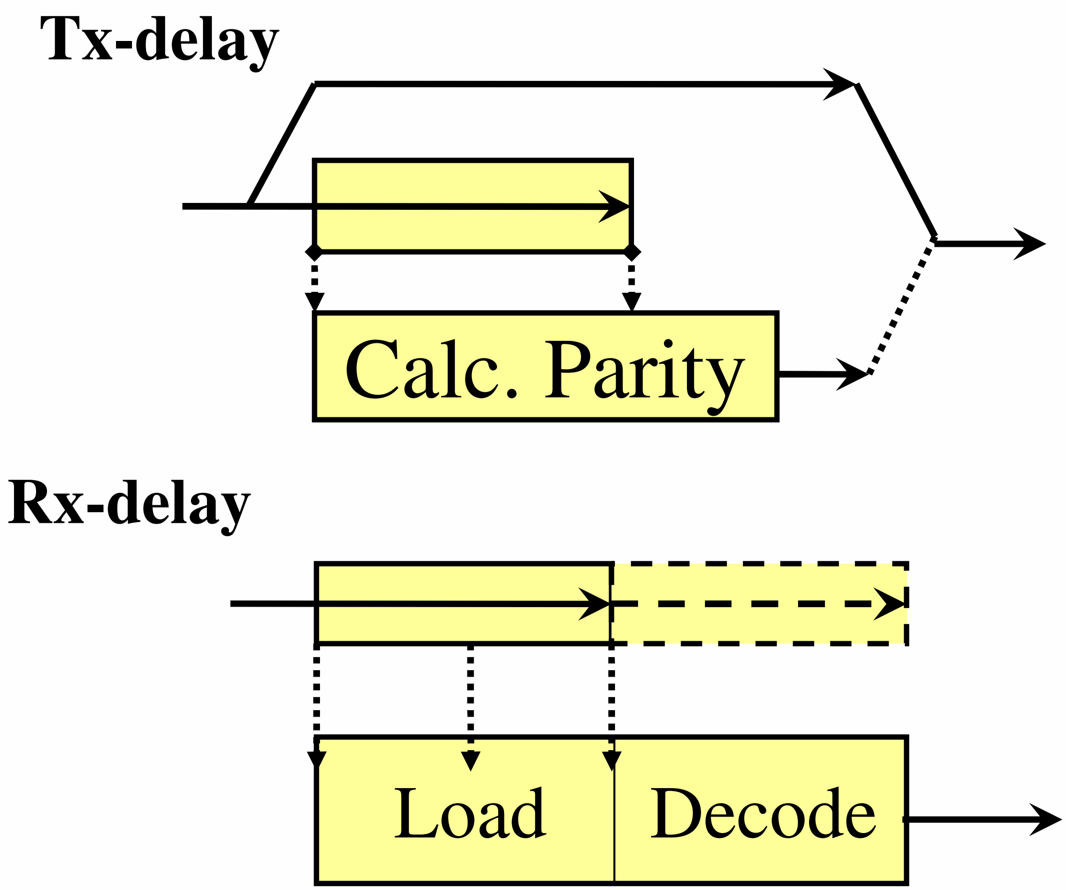
Mapping

- Turbo + Gray mapping works well, because
  - Code is long and give good coding gain anyway
  - Distance properties of individual symbols in the code is less important
  - Also more flexible with respect to
    - Modulation schemes
    - Code rate
  - Confirmed by several papers

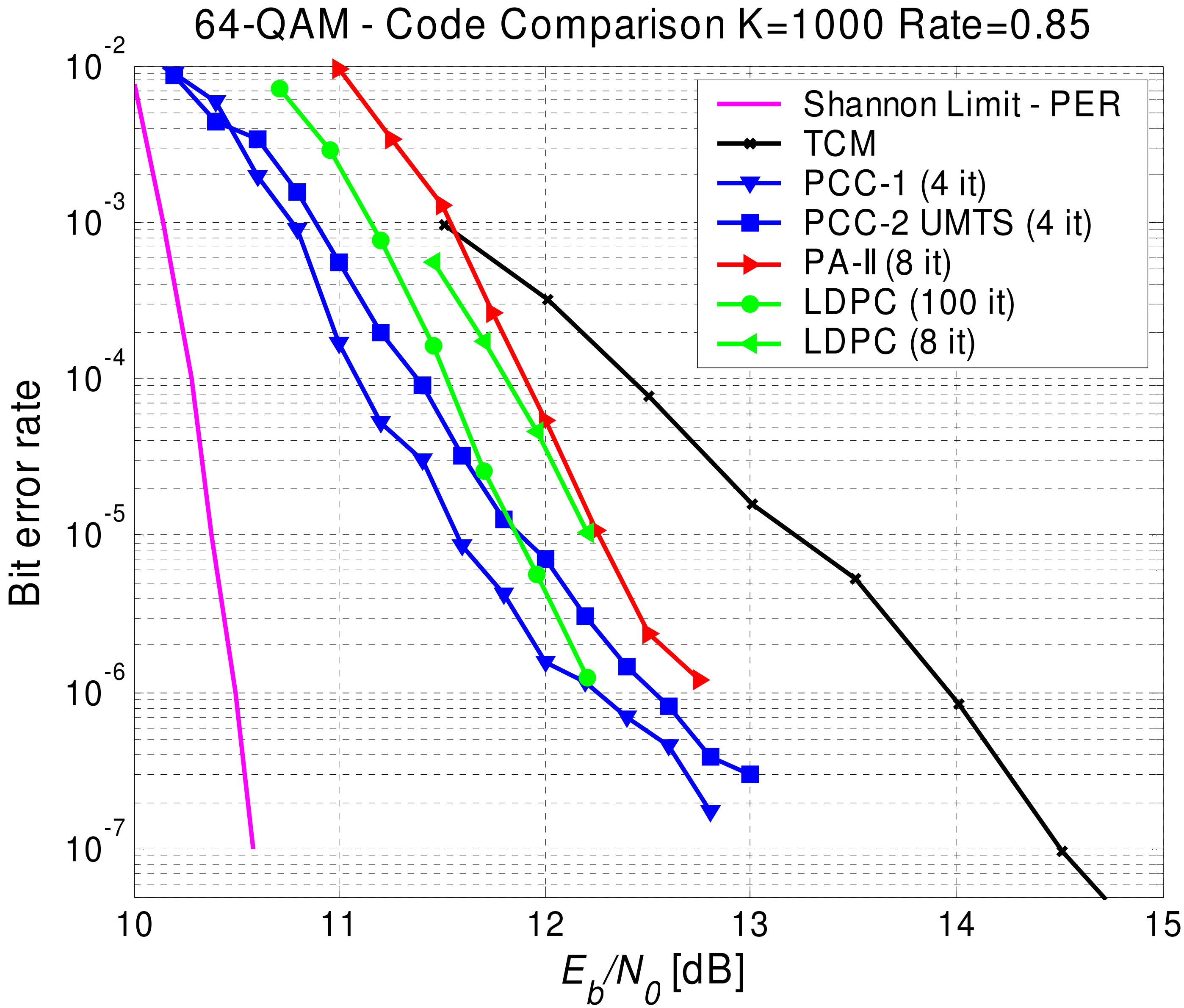
Block Length and Delay



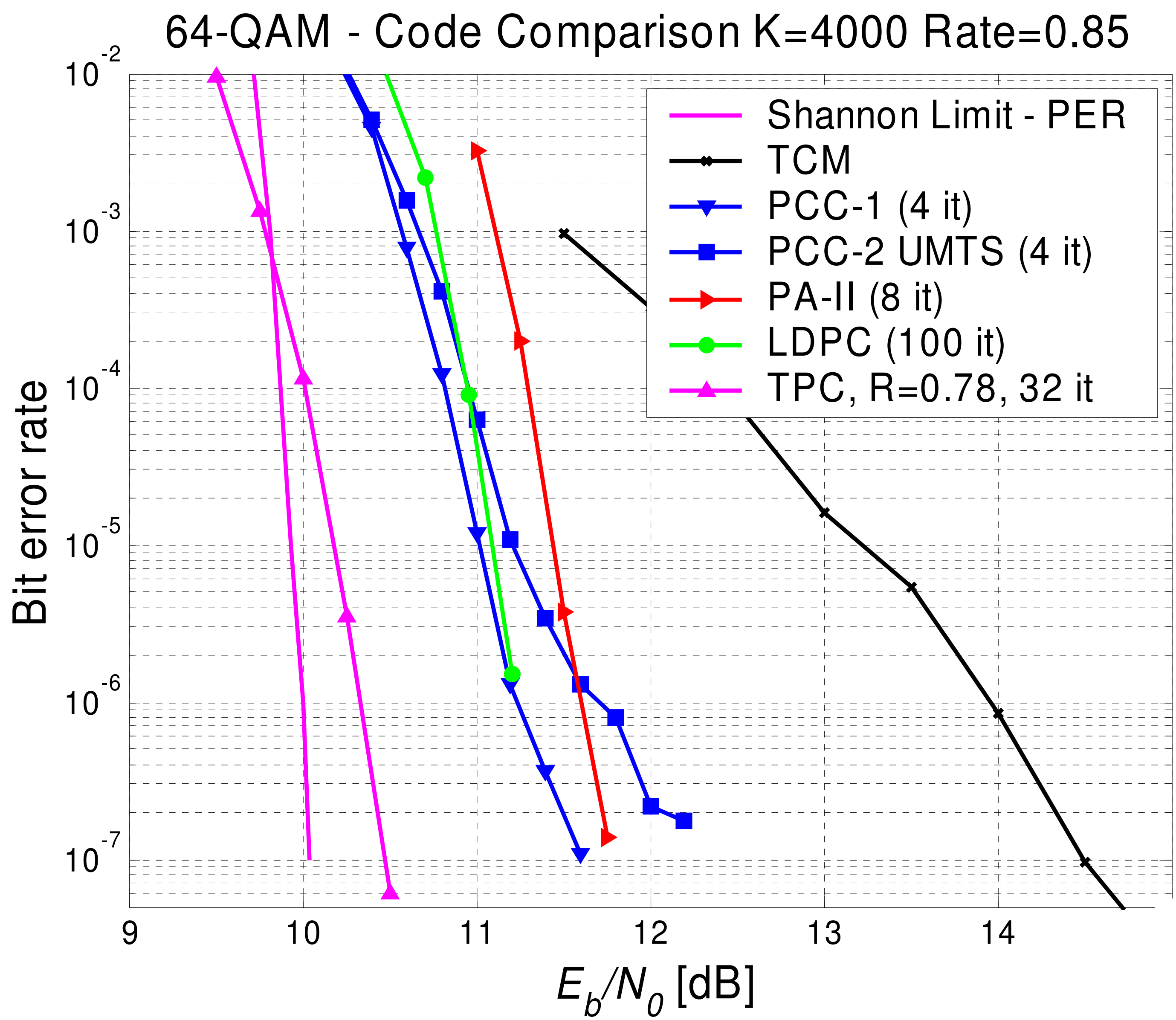
Assuming systematic encoding (block code)

Release data before new block has arrived

BER - 64QAM, *K*=1000, *R*=0.85

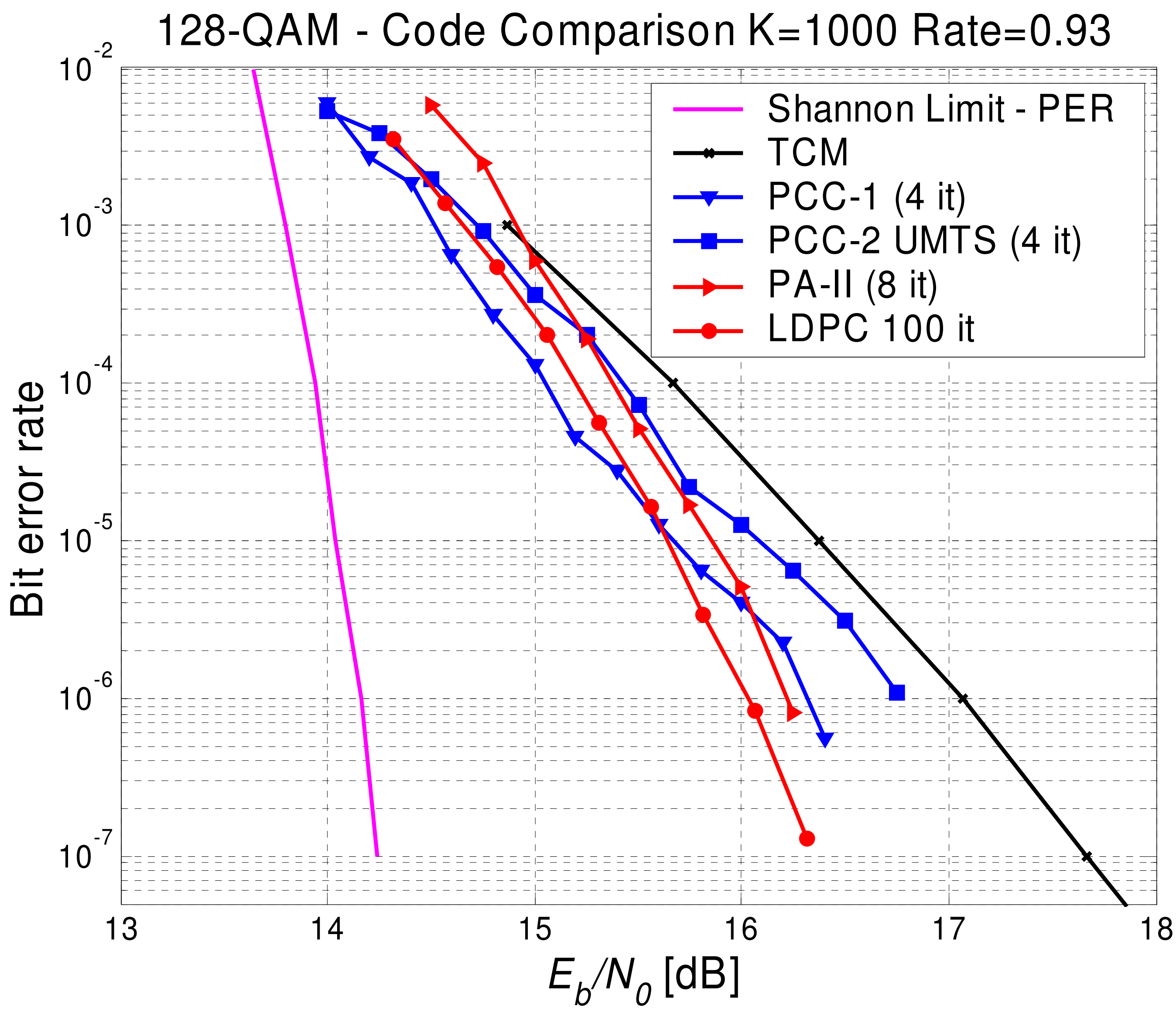


BER - 64QAM, *K*=4000, *R*=0.85



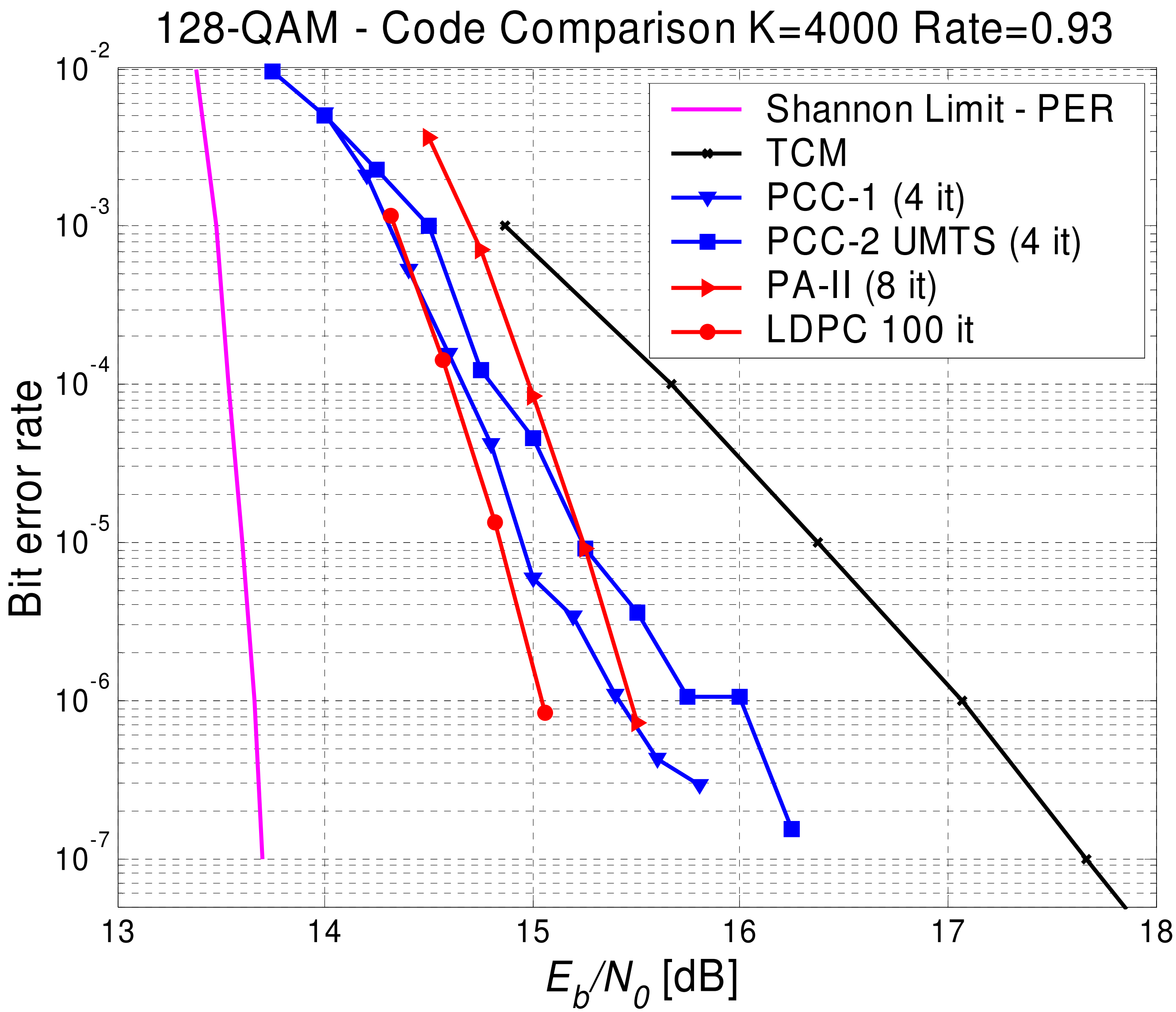
Note:
K=3249 and
R=0.78 for
TPC code

BER - 128QAM, *K*=1000, *R*=0.93



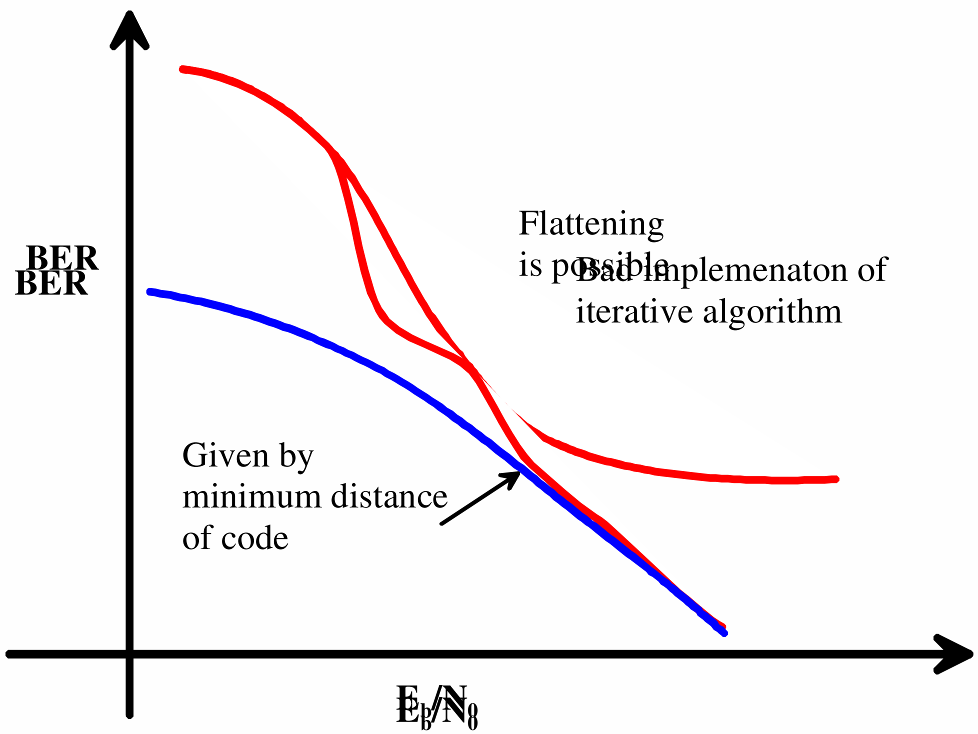
Note:
K=1458 for
PA-II

BER - 128QAM, *K*=4000, *R*=0.93

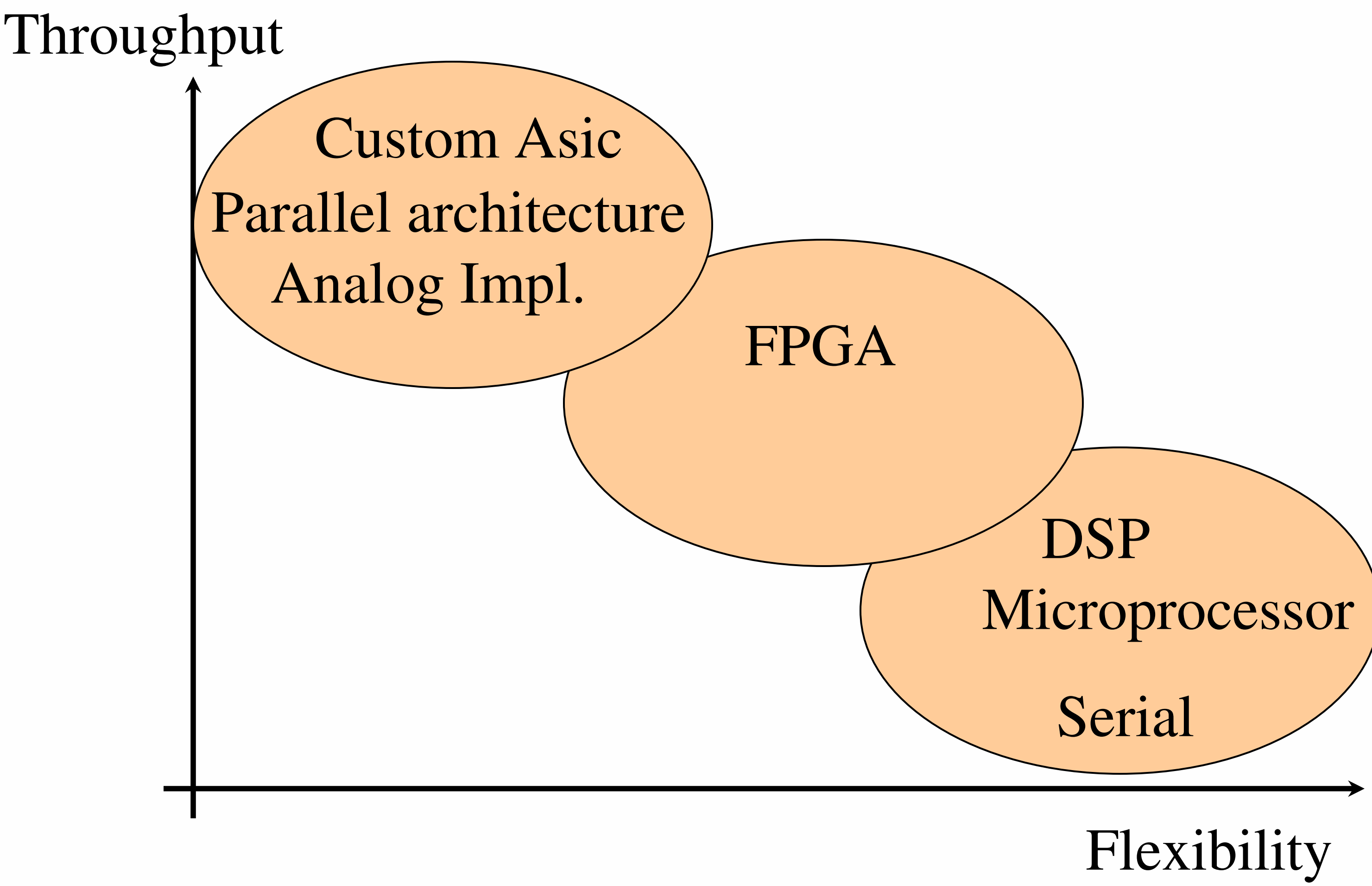


Note:
K=3645 for PA-II

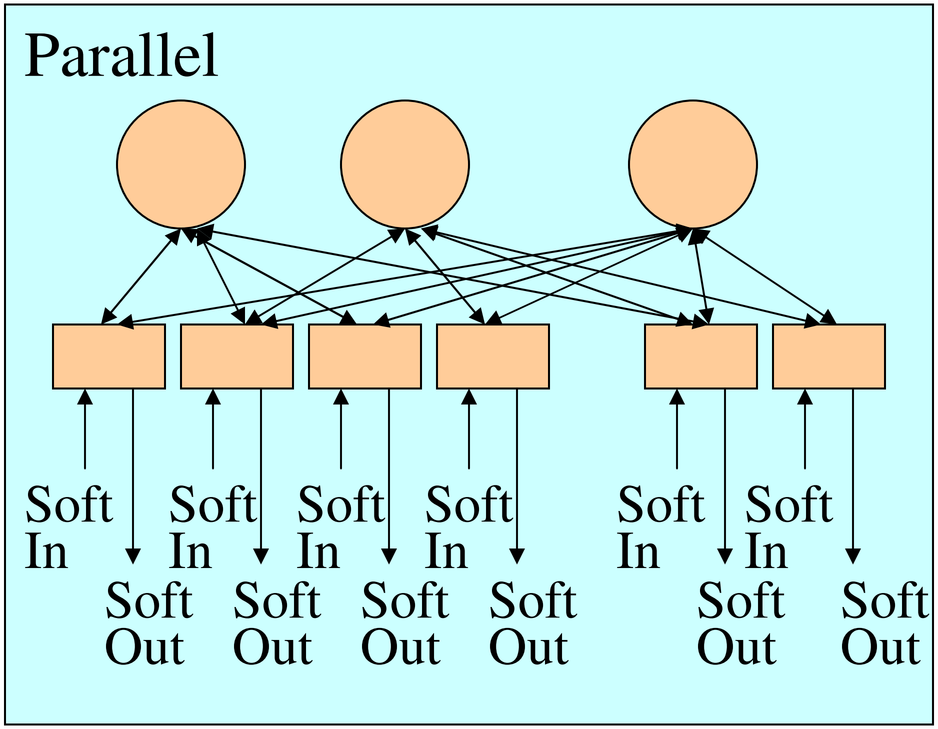
Evaluation of Iterative Decoding

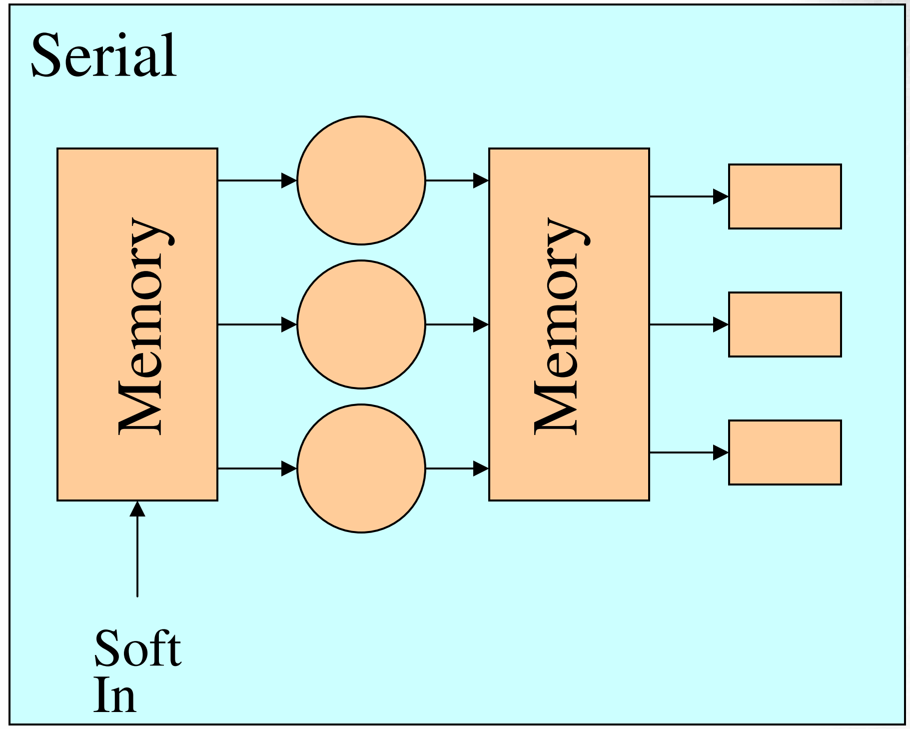


Implementation Aspects of Iterative Decoding Schemes



Implementation Example LDPC

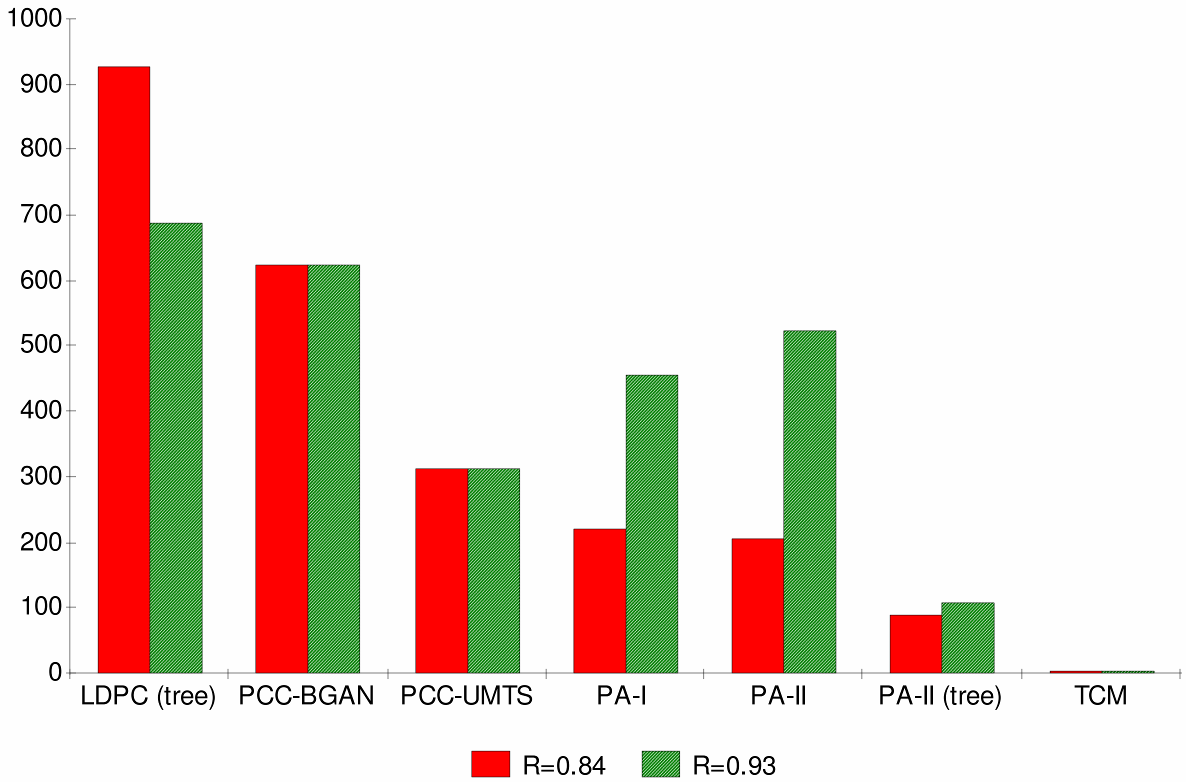




Routing limitation
Large area
Power and throughput efficient

Limitation:
- Memory and memory access
- Serial execution

Decoding Complexity of Coding Schemes



Complexity of coding schemes in # of max-log or min-sum operations pr decoded bit

Systems with Turbo/Iterative Coding Schemes

System	Coding Scheme	Typical rates
DVB-RCS	PCC code, 8 state, duo-binary	128k-3Mbit/s
IEEE802.16	Turbo Product Code	25Mbit/s
UMTS/3G	PCC code, 8 state	384kbit/s
DVB-S2	LDPC code + BCH	<40MSymb/s
BGAN	PCC code, 16 state	≤512kbit/s

- Considered also for Storage (1Gbit/s) and Optical systems (10Gb/s)

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Choice of Coding Scheme

- Coding Scheme - Chosen to Fulfil System/Application Requirements
  - Performance - BER, PER
  - Delay - Block Length
  - Spectral Efficiency - Code Rate
  - Throughput - Complexity
  - Flexibility - Scalability - Adaptivity
  - Channel Type - Mobile, Fixed, Burst Error etc

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Conclusion

- For practical systems we can not achieve the -1.6 dB limit
- “Practical limit” - depend on rate, constellation, block length
- New techniques with iterative decoding approach capacity
- Mapping not so critical for long codes
- For High Data Rates - Complexity is a Concern for real time implementation

References

For more details on the topic of this presentation, the reader is refered to the following publications (and the references therin):

- P. Orten and B. Risløw, "Theory and practice of error control coding for satellite and fixed radio systems," in Telektronikk Journal, No. 1, 2002 (can be found at www.unik.no/~porten)
- T. Flo, P. Orten, B. Risløw, "Evaluation of coding schemes for spectrally efficient low-delay radio systems," in Proceedings of 3rd International Symposium on Turbo Codes & Related Topics, Brest, France, September, 2003, pp. 387-390.